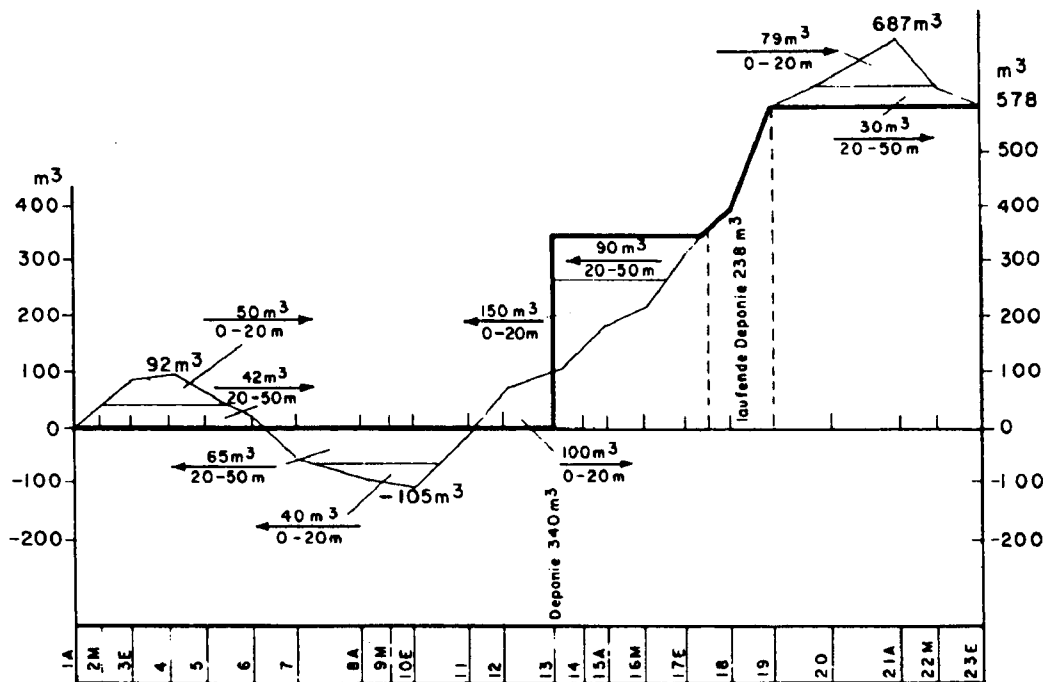




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## Mass diagram



## CHAPTER IV

### Mass Diagrams

The mass diagram can be considered to be the fourth major drawing of a road design project along with the plan, profile and cross-sections. Many of the most proficient designers and most productive contractors consider the mass diagram to be the most important diagram from their points of views. The mass diagram can be used as a tool to determine where additional work needs to be done on the geometric aspects of the design. It is also a very effective measure of the quality of a design. The generation and analysis of the mass diagram are very important concepts for the designer to master.

The earthwork computations that have been completed to this point are important because they show the designer the quantity of earth that must be excavated and how much of that earth will be placed in the embankments. As will be reviewed shortly the difference, known as the \_\_\_\_\_ represents either the excess earth material (waste) when the excavation exceeds the embankment or the shortage of earth material (borrow) when the embankment exceeds the available excavation. The mass ordinate indicates to the designer whether he or she has too much material or not enough.

At this point a number of very important questions remain to be answered.

1. How far must each cubic yard of excavated material be hauled for placement?
2. Can every excavated cubic yard of earth be hauled to a fill location economically?
3. If the excavated material cannot be economically hauled to a fill site where will it be placed? (Waste sites?)
4. How much material must be wasted?
5. If there is insufficient excavation to make an embankment, where will the additional material come from? (Borrow sites?)
6. How many cubic yards of material must be borrowed?
7. How much will it cost to haul the material?
8. If some of the excavated material can be used for surfacing how does this affect the earthwork data?
9. What areas need further design effort to reduce earthwork and costs?
10. What areas are economically designed and should not be altered?

To answer these questions and many others, another drawing is prepared and then analyzed. This is the MASS DIAGRAM. The mass diagram is a plot of the mass ordinate on the vertical scale versus the

stationing on the horizontal scale.

Reviewing the concept of the mass ordinate, if between station A and station B 500 cubic yards are excavated and 300 cubic yards are placed as fill the mass ordinate at station B is 200 cubic yards. If station A is the beginning of the project the mass ordinate for the entire project is also 200 cubic yards. At station B there is an excess of 200 cubic yards.

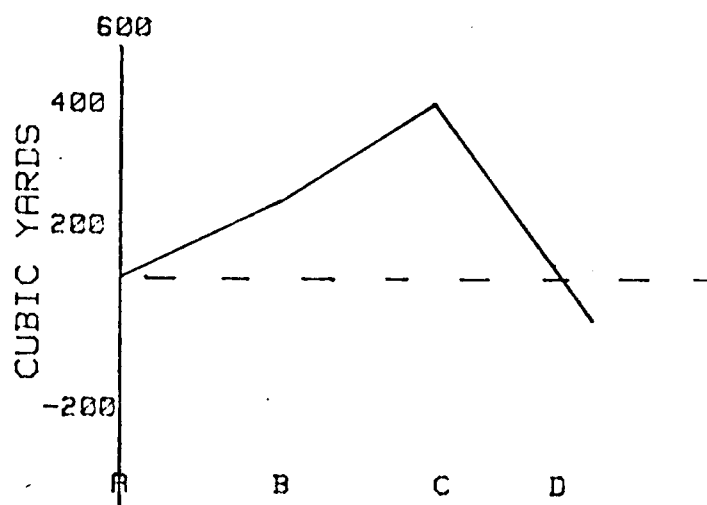
Between stations B and C 400 cubic yards are excavated and 150 cubic yards are used as fill. An additional 250 cubic yards of excess material has been accumulated. The mass ordinate, or the running total is  $(200 + 250)$  or 450 cubic yards.

Going one step further, between station C and station D 100 cubic yards are excavated but 650 cubic yards are needed for fill. Now, there is a shortage of 550 cubic yards between stations C and D. The 450 cubic yards of excess material accumulated up to station C (The mass ordinate at station C) can be used to cover part of the shortage but an additional 100 cubic yards is needed by station D. The mass ordinate at station D becomes  $(-100)$ . Earthwork shortages (excess embankments) carry a negative sign.

The mass ordinates determined to this point can be summarized in the following table:

STATION	MASS ORDINATE
A	0
B	200
C	450
D	-100

When this data is plotted the mass diagram results.



A couple of rules of thumb will assist the designer in preparing the mass diagram for a project. In the preliminary stages of the design the horizontal scale for the mass diagram should be the same as the horizontal scale for the profile. During the analysis of the mass diagram it will be easy to compare the mass diagram to the

profile if the two scales are the same. For long projects this has the disadvantage of not providing a compact mass diagram that reflects the earthwork activity over the entire project. Therefore, for a final project mass diagram designers will often select a horizontal scale that allows the plotting of the entire mass diagram on one normal sized sheet of plan paper.

The second rule of thumb is concerned with selecting the vertical scale values. Referring back to coordinate computations it was noted that some people start with large initial coordinates in order that they do not have to deal with negative coordinates. By similar reasoning some designers start with a large initial mass ordinate so that they do not have to deal with negative mass ordinates. This procedure has the distinct advantage that the datum line is eliminated. The datum line is NOT a valid balance line in most cases. If the datum line is used during the plotting of the mass diagram the designer would be well advised to erase it during the analysis of the mass diagram in order to avoid confusion.

EXAMPLE PROBLEM:

For the earthwork quantities given below compute the mass ordinates.

Station	Cut(+)	Fill(-)	Mass Ordinate
36 _____			
	700		
37 _____			
	640		
38 _____			
	530		
39 _____			
	410		
40 _____			
	360		
41 _____			
	270		
42 _____			
	130		
42+80 _____			
		20	
43 _____			
		210	
44 _____			
		450	
45 _____			
		540	

46 \_\_\_\_\_

630

47 \_\_\_\_\_

730

48 \_\_\_\_\_

820

49 \_\_\_\_\_

690

50 \_\_\_\_\_

On graph paper using a horizontal scale of 1/2 inch per station and a vertical scale of 1 inch equals 1000 cubic yards, plot the mass diagram.



Before moving on to analyzing the mass diagram spend some time studying the diagram from the example problem to become familiar with its characteristics

Ascending mass diagrams (lines going up and to the right) occur when cut exceeds fill. What type of cross-section would you expect this to occur on? road What type of profile would you expect this to occur on? \_\_\_\_\_

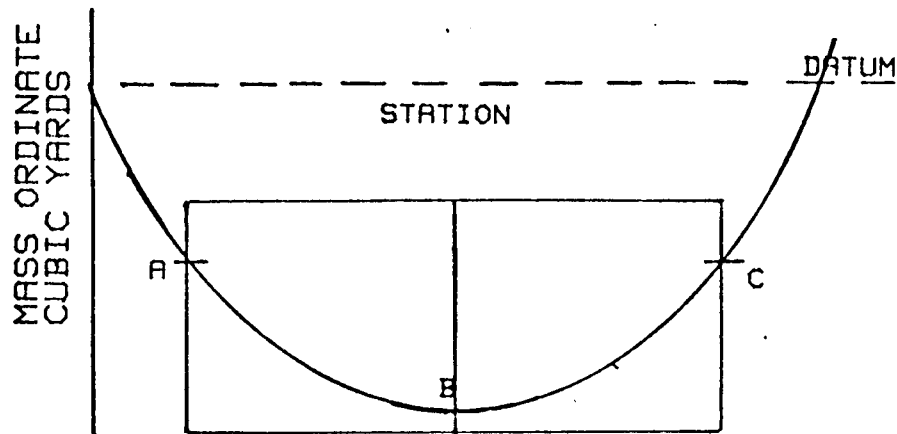
Mass diagrams where the lines go down and to the right occur when fill exceeds cut. What type of cross-section would you expect this to occur on? \_\_\_\_\_. On what type of profile would you expect this to occur? \_\_\_\_\_

A horizontal distance measured on the mass diagram represents a difference in stationing on the ground or a horizontal distance on the ground.

A vertical distance measured on the mass diagram represents \_\_\_\_\_?

A distance measured along the line of a mass diagram represents what? \_\_\_\_\_

On the figure below consider only that portion of the diagram surrounded by the box on the left.

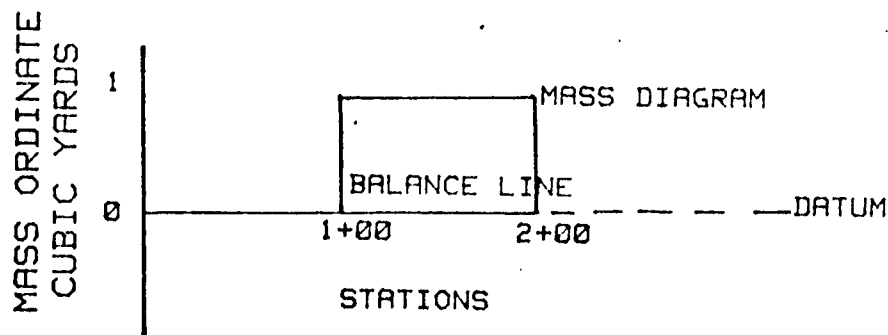


The vertical distance on the mass diagram between station A and station B is the yardage of fill generated between the two stations.

The vertical distance on the mass diagram between station B and station C is the yardage of excess cut generated between the two stations.

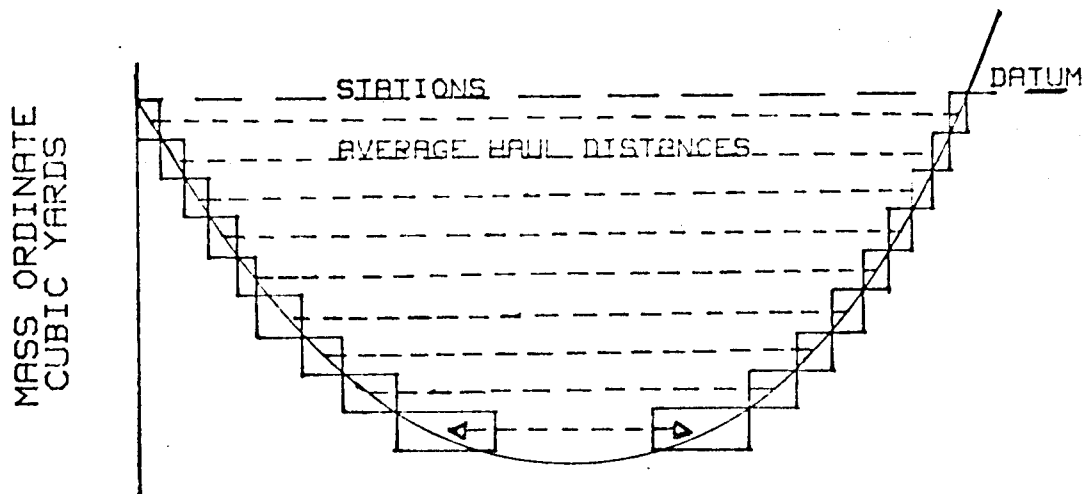
If the excess excavation between stations B and C is equal to the shortage of embankment between stations A and B then the mass diagram is said to be balanced between stations A and B. That is, all of the excess material excavated between station B and station C is placed as needed embankment between station A and station B. Since the yardage values between stations A and B are the same as the yardage values between stations B and C, the vertical distance between stations A and B and stations B and C must be equal.

characteristics of the mass diagram he or she is ready to move on to the analysis of it. In order to do that the idea of haul must be understood. Haul is the concept that deals with the cost of transporting (not excavation) a cubic yard of soil from the excavation location to the embankment location. The units of haul are stations-yards (i.e. one cubic yard hauled one station) or mile yards (i.e. one cubic yard hauled one mile.) consider the following diagram.



The simple analysis of this mass diagram shows that one cubic yard of excess excavation is generated at station 1+00 and is placed where there is a shortage of embankment at station 2+00. There is \_\_\_\_\_ stations-yards of haul. Because the vertical axis of the mass diagram has units of cubic yards and the horizontal axis has units of stations, the haul could have been determined from the area within the loop. Any mass diagram can be broken down to a series of rectangles that account for each cubic of excess soil material. The area of each rectangle is one cubic yard times the distance that that cubic yard is transported in stations. If the area for all of the individual cubic yards are totaled together, the total haul for the balanced loop would be determined. Thus it is considerably easier to simply measure the area within the loop to determine the total cubic

yards of haul. For example:



Alternate computational methods of determining the haul are available. However, they are tedious and are usually destined for computer solution.

The next question that should come to mind is where does the balance line go on the mass diagram. To answer this question the concept of LIMIT OF ECONOMIC HAUL (LEH) must be understood. The LEH is the maximum distance that soil material can be hauled before it is more economical to waste that material and borrow additional yardage to complete the fill. The limit of economic haul is computed from the cost of borrow and the cost of haul.

Borrow cost (\$/cubic yard)

Limit of economic haul = \_\_\_\_\_

Haul cost (\$/station-yard)

The limit of economic haul is determined then, by the cost of borrow in the area of the shortage of embankment not the cost of borrow in the area of the excess excavation. If a borrow area is not

conveniently located near the area where the material is needed, the cost of borrow must include both the cost of the borrow material and the cost of hauling that material. A mass diagram may have several different LEH values depending on the nature of the soil material on the project. In general, however, a relatively simple computation will determine the length of the balance line.

The analysis of the mass diagram begins by installing the balance lines on the mass diagram. This is done by taking a straight edge to the top (or bottom) of each loop as appropriate. Slide the straight edge down (or up) taking care to keep it horizontal. When the straight edge spans the distance of the LEH and intersects the mass diagram, the balance line can be drawn in.

EXAMPLE PROBLEM: Given the mass diagram from the previous example problem and,

the unit cost of borrow is \$3.60 per cubic yard

the unit cost of haul is \$0.30 per station-yard.

1. Determine the limit of economic haul.
2. Plot the balance line on the mass diagram.

At what stations does the balance line intersect the mass diagram?

Label the intersect point (balance point) on the left as A and the one on the right as B.

Examining the mass diagram developed to this point, from stations 36+00 to point A (about station \_\_\_\_\_) excess excavation is being generated. It is too far to haul the excess soil to the other end of the job (as determined by the LEH,) therefore, the material becomes \_\_\_\_\_.

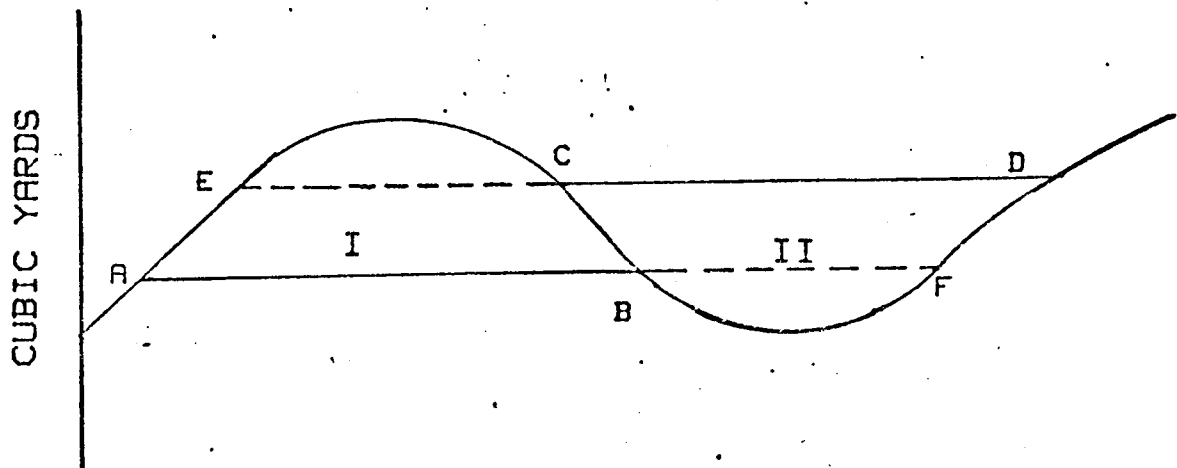
From the right hand balance point, B (about station \_\_\_\_\_) to station 50+00 there is a shortage of embankment material. Again the balance line shows that it is too far to haul the material in from the other end of the job. In this case the material must be \_\_\_\_\_.

If you go over any completed mass diagram every cubic yard must be accounted for either as waste, borrow, or within a balanced loop.

With the loop now balanced the total amount of haul can be determined. Again, this is most easily done by simply measuring the area within the loop. This can be done by any of the methods of measuring or computing areas discussed under earthwork computations.

EXAMPLE PROBLEM: Determine the haul in both station-yards and mile-yards for the example problem above.

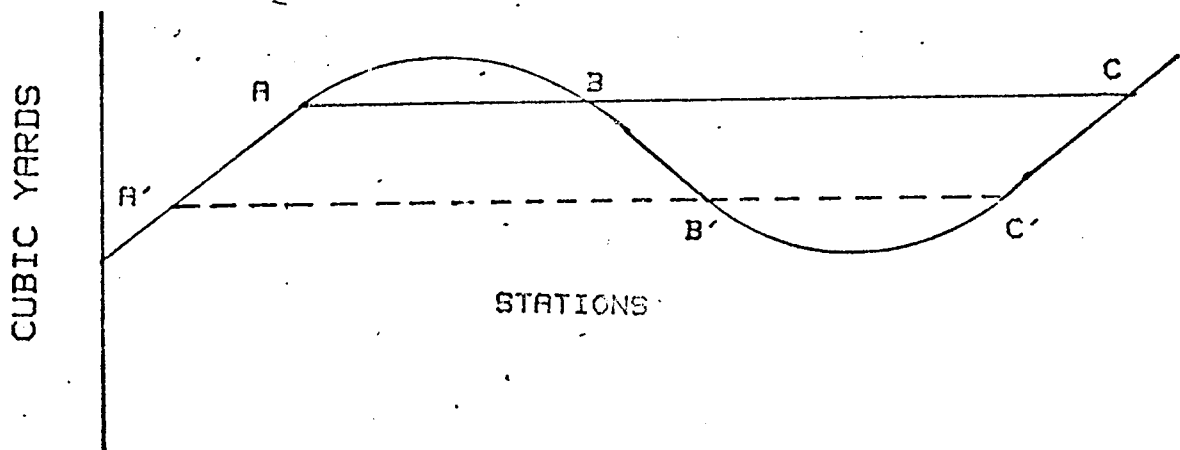
The balancing of mass diagrams is not often made up of a series of single loops. Consider the following example:



If the mass diagram is balanced as previously discussed the balance line for loop I will be located at AB and for loop II at CD. The problem that develops is in the area of the mass diagram between C and B. Since the mass diagram is descending to the right there is a shortage of excavation between C and B. The balance line for loop I, line AB, indicated that the material will come from approximately stations A to E. The balance line for loop II, line CD, indicates that the material will come from approximately stations BF. The result would be twice the amount of material required would be delivered between station C and B. Since this is obviously not a workable solution another location for the balance line must be determined. A general rule results from this situation.

Balance lines for adjacent loops on the mass diagram cannot overlap.

When the mass diagram consists of two loops the question becomes where does the balance line fall and why? Consider the following mass diagram:



The balance lines have been located such that balance line BC and balance line A'B' are at the limit of economic haul. That is to say that the final balance line can go no further up than lines AB and BC and no further down than lines A'B' and B'C'. The basic logic that directs where the balance line should go calls for minimizing the cost of haul. This means minimizing the haul or the area within the loop. This can be done by minimizing the longest haul distance that any material must be transported. To do this the balance lines should be positioned such that the distance AB equals the distance BC. This can best be done graphically using a scale. Similar reasoning can be applied to cases where there are four, six or any EVEN number of loops. The general rule for EVEN numbers of loops is:



Where the placement of balance lines on a mass diagram results in an even number of loops one balance line should be drawn in such that the total length of the even numbered loops equals the total length of the odd numbered loops.

$$L_1 + L_3 \dots = L_2 + L_4 \dots$$

When the mass diagram balances out with three loops the rule for locating the balance line is changed only slightly. The overall goal for balancing is the same, to minimize the cost of haul, to minimize the area under the loops. The rule for balancing a mass diagram with an ODD number of loops is:

Where the placement of balance lines on a mass diagram results in an odd number of loops, one balance line should be drawn in such that the total length of the odd numbered loops equals the total length of the even numbered loops plus the limit of economic haul.

$$L_1 + L_3 \dots = L_2 + \dots + LEH$$

By now the reader should be well aware of the fact that the datum line used for drawing the mass diagram may or may not be a balance line. Again the designer is warned to erase the datum line after

preparing the mass diagram and before drawing in the balance lines. This will prevent the possible confusion of the datum line and the balance lines.

Regardless of the number of loops within the balance points and regardless of whether the loops are above or below the balance line the haul can be determined from the area within the closed loops.

A list of procedures involved in balancing a mass diagram is as follows:

1. Plot the mass diagram from the earthwork and mass ordinate information.
2. Determine the limit of economic haul (LEH) for each portion of the mass diagram where there is a shortage of embankment.
3. Tentatively draft in the balance lines at their maximum distances but not to exceed the LEH's.
4. Noting where multiple loops result, rebalance the diagram using either the even loop or odd loop rule as appropriate.
5. Measure the area within the loops to determine the haul in station-yards.
6. For those portions of the mass diagram that fall outside of the loops determine whether each section requires borrow material or generates material to be wasted.

The next step in analyzing the mass diagram is sometimes

confusing to designers with limited experience. To gain a full understanding of the procedure and its importance you should recall the earthwork computation sheet. After the volume of excavated material was computed it was multiplied by the shrink/swell factor. The volume before the multiplication was the volume of soil actually excavated from between the two cross sections. This was computed because the contractor is paid for the actual amount excavated not for the expanded or shrunk amount that may result after handling. However, the mass ordinate, and therefore the mass diagram, is determined from the modified earthwork volumes.

Just as the contractor is paid for the actual excavated quantity he is also paid to haul the actual excavated quantity, not the shrunk or expanded amount. Therefore, the haul amount in station-yards must be converted back to the actual excavated yardage. This is done simply by dividing the total haul by the shrink/swell factor for that material at the excavation site.

In a similar way the borrow and waste yardages must be converted to actual excavated yardages.

After the mass diagram is completely balanced (but usually before the determination of haul) the designer is in a position to utilize it as a guide to improve the design. The location of excessive amounts of waste or borrow can be determined by observation. The designer can then determine whether additional design effort is justified at those locations.

One of the overall goals of design is to minimize cost which translates into minimizing both excavation and haul. The designer must

be extremely leery of redesigning a portion of the roadway simply to minimize the area under a loop. While the reasoning behind this is simple it is not always seen even by experienced designers. That reasoning is simply do not design in a great deal of excavation just to eliminate some haul. The cubic yards of excavation are usually several times more expensive than the mile yards of haul that are eliminated. Avoid increasing excavation just to decrease haul. A total cost reduction must be achieved in order to justify the design changes.

The mass diagrams that have been developed up to this point have the point of view of the designer. That is, the procedure of modifying the excavated quantity provides the designer with correct volume of material that will be available for constructing embankments. The procedure does have the disadvantage of having to readjust the borrow, waste, and haul figures to determine the volumes that the contractor will actually be responsible for excavating and hauling.

The most desirable alternative is to prepare the mass diagram based on the earthwork figures generated by modifying the embankment. As was seen in previous chapters, these computations are no more difficult than in the first case. The preparation of the mass diagram is identical, as has already been discussed. On any project where more than one shrink/swell factor is used because of changing soil types (the most common situation) the designer MUST prepare a mass diagram based on modifying the excavation. This first mass diagram shows the designer where each yard of embankment originates from, and therefore, what shrink/swell factor must be applied to the

embankment volume at each station. When these factors have been determined at each station, the earthwork figures can be recomputed and the revised mass diagram prepared.

The analysis of the revised mass diagram is much simpler because excess excavation, waste, borrow and haul quantities can be interpreted directly. They do not have to have the shrink/swell factors adjusted out. In the practice problem that follows, the student should note that the final dollar costs come out to be the same as in the previous practice problem.

The advantages and disadvantages of modifying either the excavation or embankment are numerous. The major difference is that adjusting the embankment has considerably more computational work than adjusting the excavation, although the analysis of the mass diagram is much simpler. Either system will result in the same decisions if the analysis of each is properly carried out.

There are several specialized problems that develop on mass diagrams that the designer must be able to handle. These include cross haul, stockpiles, borrow pits and waste areas (both on and off line) and surfacing spreads.

Cross haul gets its name from the fact that in this situation the haul is being made across a balanced loop. Consider for example the following profile and mass diagrams.

As shown on the profile between stations R and S, the native soil condition is a soft bog area. It has been suggested that a heavy rock fill be used in this area to stabilize the bog. An adequate supply of rock exists between stations U and V. Between stations S and

supply of rock exists between stations U and V. Between stations S and T there is a sandy silt material that is to be cut away. While this sandy silt is an adequate material for the fill from station T to station U, it is not acceptable material for the fill from stations R to S. In this situation, the soil materials on the ground force some special construction procedures that must be accounted for in the design and in the cost estimate.

